Electrodynamics of accelerated charges or Why electron does not radiate in Rutherford's atom

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It is shown, that the radiation of the charge, moving with uniform acceleration or uniformly moving round a circle and also freely moving in a gravitational field, contradicts the principle of equivalence. It is also shown, that the interaction of the charges, moving with uniform acceleration or uniformly circling, which has been calculated within the framework of classical electrodynamics, leads to the violation of laws of conservation of energy, impulse and angular momentum. We have offered a method in which way to conform electrodynamics to the principle of equivalence. So in the electrodynamics, which has been conformed in such a way, all the mentioned violations of the laws of conservation are automatically removed and the stability of Rutherford's atom is explained. It is shown that the changes, which we have brought into the electrodynamics, do not contradict the results of experiments.

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1 The violation of the principle of equivalence

The principle of equivalence which is the base of the general theory of relativity reflects the close connection between inertial coordinate system K with uniform gravitational field and noninertial coordinate system K' moving with uniform acceleration in empty space. According to this principle any physical process proceeds absolutely identically in the both systems. In other words, if we imagine the system K as a closed laboratory, which is at rest on the Earth, and the system K' as an identical laboratory, moving with acceleration g in a distance of gravitating masses, and the sizes of the laboratories are such chosen, that it will be possible to neglect the nonuniformity of the gravitational field in system K, then the observer, who is in one of these laboratories and who does not have any connection with the external world, i. e. without having any possibility to look out of its limits, could not make any experiment inside the laboratory in order to find out in which of them he is, i. e. he could not define the character of his motion. That can be explained by the fact, that in system K' during the uniform accelerating the field of inertial forces appears. The action of this field does not differ from the action of the gravitational field.

It is easy to make sure, that the mechanical processes will proceed identically in both laboratories. In fact, free bodies in every laboratory will move with acceleration g, identical pendulums will oscillate with equal periods, etc. Thus, systems K and K' are equivalent in respect of mechanical processes.

Einstein spread the equivalence of systems K and K' on all physical processes without exceptions, having formulated the principle of equivalence, according to which, not only mechanical, but also any physical processes have to proceed identically in systems K and K'.

But we can point to the process which violates the principle of equivalence. It is the radiation of charges. It is easy to make sure, that with the help of charges it is possible to differ system K from system K'. In fact, let us place the charges in both laboratories. The charge in the laboratory K' has to radiate, so it is moving with acceleration. The observer who is in this laboratory can register this radiation having placed, for example, a charge into the water. Then a part of the radiating energy will be absorbed by the water and will warm it up. On measuring the temperature of the water the observer will be to register the radiation.

We won't discuss the technical details how to carry out such an experiment. It is enough that it is a principal possibility to find out the radiation in immediate proximity to the charge.

In laboratory K a motionless charge will not radiate. Thus, the observer in every laboratory very easily can define the character of his motion.

Therefore the principle of equivalence is violated.

Now we will consider uniformly rotating coordinate systems.

We can imagine one of these systems (we mark it as S') as a disk, revolving with constant angular velocity on its axis. In accordance with the principle of equivalence the noninertial coordinate system S', in which a field of centrifugal forces of inertia exists, can be considered as an inertial system K with an uniform gravitational field.

Let us consider two charges: one of them is on the disk, i. e. rotates with this disk. The second one is at rest at system K. In the first case the charge has to radiate and in the second one it does not. It has been already told how to differ a radiating charge from a charge, which does not radiate. And what is more, a braking force of radiation friction f must act on uniformly moving round a circle charge, which action one can observe at an as short as we want distance from the charge.

Therefore, in this case the principle of equivalence is violated.

So far as we have mentioned the radiation force of friction, it is necessary to point out another problem, which was first remarked by M. Born. By motion of the charge with uniform acceleration, force f turns into zero. That leads to the violation of energy balance. Indeed $f = \dfrac{2e^2}{3c^3} \cdot \dddot{r}$, where $\dddot{r}$ — the third derivative from a coordinate. If the motion firmly accelerates, $\ddot{\vec{r}}$=const and $\dddot{\vec{r}}$=0. Therefore, an uniformly accelerating charge radiates without any losses of energy. That contradicts the law of conservation of energy.

We will return to laboratories K and K' again. We will change the character of their motion. Let laboratory K', which is in empty space far from gravitating masses, move uniformly and straightforward and let K freely move in a gravitational field. There will be state of weightlessness in both laboratories. According to the principle of equivalence, in this case systems K and K' are also equivalent. I. e. as in the last instance, the observer, who is in one of the laboratories and does not have any connection with the external world, could not make any experiment inside the laboratory in order to find out the character of his motion.

But we will place a charge in each of these laboratories again. In laboratory K charge must radiate, for it moves with acceleration under the action of the gravitational field. The observer in this laboratory will be able to register the radiation. In laboratory K' the charge will not radiate.

Therefore, in this case the principle of equivalence is not fulfilled.

2 The violation of laws of conservation in classical electrodynamics

By means of not complicated calculations it can be shown, that there is a violation of laws of conservation of energy, impulse and angular momentum in classical electrodynamics. And the violation of these laws arises when we consider the charges moving with uniform acceleration or uniformly moving round a circle — i. e. those charges, the electrodynamics of which violates the principle of equivalence. One can make sure in that, having calculated the interaction of such charges.

Let charge e move arbitrarily. Its field $\vec{E}$ is determined with the following expression

$$\vec{E} = \frac{e\left(1-\frac{v^2}{c^2}\right)}{\left(R-\frac{\vec{R}\,\vec{v}}{c}\right)^3}\left(\vec{R}-\frac{\vec{v}}{c}R\right) + \frac{e}{c^2\left(R-\frac{\vec{R}\,\vec{v}}{c}\right)^3}\left[\vec{R}\left[\left(\vec{R}-\frac{\vec{v}}{c}R\right)\dot{\vec{v}}\right]\right] \tag{1}$$

All the quantities in the right part are taken at the moment t', which precedes the moment t of the observation and which can be obtained from the equation

$$t' + \frac{R(t')}{c} = t$$

where $R(t')$ is a distance from the charge to the point of observation and $\vec{v}$ and $\dot{\vec{v}}$ are velocity and acceleration of the charge. The field consists of two parts. The first term describes Coulomb's field of the charge, and the second one describes the field of radiation.

We will consider two nonrelative charges e and $-e$, connected with a pivot, which length is l. These charges are moving with acceleration $\dot{\vec{v}}$, directed along the pivot (fig. 1). Let us define Coulomb's field $\vec{E}_{k1}$, which the charge e creates in the point C, where charge $-e$ is. At the moment t' the charge e is in point A. Vector $\vec{R}(t')$ is equal to the length of segment AC by its module. Vector $\dfrac{\vec{v}}{c}R$ is equal to the length of segment AB (fig. 1). Point B is a position, in which charge e could be at the moment t, as if it were moving from point A uniformly with velocity $v(t')$ during the period of time $t-t'=\dfrac{R}{c}$. But the charge e is moving with acceleration $\dot{\vec{v}}$. That is why at the moment t it will be in point D. And segment $AD = v\tau + \dfrac{\dot{v}\tau^2}{2}$, where $\tau = t - t'$.

Therefore the Coulomb's field $\vec{E}_{k1}$ is equal to

$$E_{k1} = \frac{e}{\left(l-\frac{\dot{v}\tau^2}{2}\right)^2}\,.$$

For $v \ll c$, then $R(t') \approx l$, $\tau \approx \dfrac{l}{c}$, then $E_{k1} = \dfrac{e}{\left(l-\frac{\dot{v}l^2}{2c^2}\right)^2}$.

Therefore, force $\vec{F}_1$, acting on the charge $-e$, is equal to

$$F_1 = \frac{e^2}{\left(l-\frac{\dot{v}l^2}{2c^2}\right)^2}\,.$$

In the same way we will determine field $\vec{E}_{k2}$, which the charge $-e$ creates in point D, where the charge e is. At the moment t' the charge $-e$ will be in point K (fig. 1). The segment $KL = v\tau$. The segment $KC = v\tau + \dfrac{\dot{v}\tau^2}{2}$.
So field $\vec{E}_{k2}$ is equal to

$$E_{k2} = \frac{e}{\left(l + \frac{\dot{v}l^2}{2c^2}\right)^2}.$$

The force $\vec{F}_2$, acting on the charge e is equal to

$$F_2 = \frac{e^2}{\left(l + \frac{\dot{v}l^2}{2c^2}\right)^2}.$$

Thus, forces $\vec{F}_1$ and $\vec{F}_2$ are not equal by the quantily. Their resultant $\Delta\vec{F} = \vec{F}_1 + \vec{F}_2$ is

$$\Delta F = \frac{2e^2\dot{v}}{c^2 l}$$

and is directed along the vector $\dot{\vec{v}}$.
The forces of the interaction of the charges, which are specified by the second item in (1), are equal to zero in this case.

In the case, when vector $\dot{\vec{v}}$ and the axis of the pivot make angle α, the forces $\vec{F}_1$ u $\vec{F}_2$ are not equal by the quantity and do not lie on the same straight line (vectors $\vec{R}$ and $\dfrac{\vec{v}}{c}R$ of the charge e are shown in fig. 2).
The projection of their resultant $\Delta\vec{F}$ on axis x is equal to

$$\Delta F_x = \frac{e^2\dot{v}}{c^2 l}(2\cos^2\alpha - \sin^2\alpha)$$

and on axis y

$$\Delta F_y = -\frac{3e^2\dot{v}}{2c^2 l}\sin 2\alpha.$$

The forces of the interaction of the charges, specified by the second item in (1), are not equal to zero in the ammount in this case either. The projection of its resultant $\Delta F'$ on axis x and axis y are equal

$$\Delta F'_x = \frac{2e^2\dot{v}}{c^2 l}\sin^2\alpha;$$

$$\Delta F'_y = \frac{e^2\dot{v}}{c^2 l}\sin 2\alpha.$$

It is quite evidently, that the presence of the resultant $\Delta\vec{F}$, depending on the orientation of the pivot and acting on the electrically neutral system, contradicts the laws of conservation of energy and impulse. Within the framework of classical electrodynamics it is impossible to compensate this force.

We will consider the following instance. Let cylinder of radius r the forming d revolve on its axis. We should choose quantaties r and d, so as $2\pi r = 0,01\ d$. At the ends of the forming we place charges e and $-e$ (fig. 3). Let linear velocity of the charges be equal to $v = 0,01\ c$. In order to calculate Coulomb's field $\vec{E}_k$, which the charge e creates in point A, where charge $-e$ is, it is necessary to find out the preceding position of charge e at moment t' — as in the previous instance. It is evidently, that for this case the period of time $t - t'$ is equal to the period of revolution of the cylinder. So, at the moment t' the charge e will be in point B, where it is also at the moment t. Vectors $\vec{R}$, $\dfrac{\vec{v}}{c}R$ and $\vec{R} - \dfrac{\vec{v}}{c}R$ will be directed as it is shown in fig. 3 ($BP = 2\pi r$). The field $\vec{E}_k$ in point A will be equal $E_k \approx \dfrac{e}{d^2}$ and will direct along the segment PA.

That is why Coulomb's force, acting on the charge $-e$, has got the component on the tangent to the point A. The quantity of this component is equal to $F = 0,01\ \mathrm{F}_k$ where F_k is Coulomb's force.

The force, acting on the charge $-e$, specified by the second item in(1), will be directed perpendicularly to the velocity of the charge $-e$.

In the same way we will calculate the effect of charge $-e$ on charge e. There fore on the cylinder under review, which is a closed system, the rotational moment $M = 0,02\ \mathrm{F}_k r$ is acting. That contradicts the principles of conservation of energy and angular momentum.

It is easy to show, that if the velocity of the rotation of this cylinder is diminished twice, Coulomb's interaction of charges e and $-e$ will lead to the braking of the cylinder.

3 Rutherford's atom

One of the most important problems of classical electrodynamics is the problem of the stability of Rutherford's atom. As it is known, the planetary model of atom, offered by Rutherford, has a principal shortcoming — it was unstable. According to classical electrodynamics, an electron, moving round a circular orbit, has to radiate and as a resultat it will fall down on the nucleus. Bohr's postulates, which had appeared soon after that, did not make clear this problem — the existance of allowed orbits also contradicted classical electrodynamics.

The quantum mechanics does not explain either, why electron in a stationary orbit does not radiate. The radiation of the electron, when it is going from one allowed orbit to another one, is not connected with the radiation of an accelerated electron anyhow. Atom can be in an excited state rather long, but an accelerated electron must radiate constantly. And what is more, in quantum mechanics the concept of movement with acceleration is not considered at all. The reference to in applicability of classical electrodynamics to atomic objects is quite unfounded. In fact, electron is held by forces, which submit to principles of electrodynamics. Then, why don't these principles spread on motion of electrons?

Thus, the problem of the stability of atom remains to be unsolved up to now.

4 Do inertial coordinate systems exist?

In classical electrodynamics motion of charges is considered in inertial coordinate systems. These systems are defined in the following way: if any forces do not act on the body, or the sum of acting forces is equal to zero, then there are such coordinate systems, relative to which the body is moving without any acceleration. These systems are named inertial (The first Newton's law). Inertial systems are moving uniformly and straigtforward relatively to each other, so the acceleration of the body has identical value in any of these systems, i. e. acceleration is absolute quantity.

But how can we realise inertial coordinate system in practice? Obviously, in order to realise it, it is necessary to take a body, on which any force does not act at all, or their sum must be equal to zero, and then to find a coordinate system, relatively to which this body will be moving without any acceleration. That system will be inertial.

But it is impossible to isolate a body from any action of external forces. Such isolation would mean, that there is only one body in the Universe. Thus, we can only try to bring the sum of external actions to zero. But how can we find out, that the sum of forces, acting on a body, is equal to zero? We can do it only in the following way — to define the acceleration on the body relatively to inertial coordinate system. If the acceleration is equal to zero, then the sum of the forces is equal to zero.

So we have got an exclusive circle; in order to determine an inertial coordinate system we have to choose a body so as the sum of acting forces on it is equal to zero; but we need to have an inertial coordinate system in order to determine equality of this sum of external forces to zero.

Thus, we have got a real difficulty when we try to realize the inertial coordinate system in practice.

In principle there is a possibility in classical electrodynamics to realize the inertial coordinate system. In fact, according to electrodynamics an accelerated charge must radiate. The intensity of radiation (in nonrelativistic case) is defined with an expression

$$I = \frac{2e^2w^2}{3c^3} \tag{2}$$

where e is a quantity of the charge, and w is its acceleration. Equations of electrodynamisc are true in inertial coordinate systems.

That is why, if a charge moves with acceleration, it radiates, if its acceleration is equal to zero, there is no any radiation. Radiation is an absolute quantity. It is impossible to create or to destroy it by any choose of any coordinate system. So the acceleration, which is undoubtedly connected with radiation, has to be an absolute quantity. Therefore, if some charge does not radiate, the system, which is connected with it, will be inertial.

In classical physics it is supposed, that equations of electrodynamics are true only in inertial coordinate systems. It is easy to make sure, that such a point of view is not founded enough. Indeed, the equations of Maxwell were received as a result of the generalization of experimentals. So far as there already were distinguished inertial coordinate systems, the equations of Maxwell were brought to these systems. But we do not have any grounds to affirm, that the experimentals, in which electromagnetic phenomens are studied and which are carried out in inertial coordinate systems and the results of the same experiments, which are carried out in an uniformly accelerated coordinate system, will be different. And if the results of these experiments in these systems are identical, in will mean, that the equations of Maxwell will be also true in uniformly accelerated systems.

Will Coulomb's law be true in uniformly accelerated systems? In other words, if one can measure the interaction of motionless charges in a laboratory, which moves with uniform acceleration, then can this interaction be described by Coulomb's law? In particular, will the forces of the interaction of the charges be on the same straight line? Obviously, that we can answer this question only on the grounds of the experimental. According to classical physics, Coulomb's law is not true in uniformly accelerated coordinate system, for the charges, which are at rest in this system, move with acceleration in inertial coordinate systems, and the electromagnetic field of such charges is described by means of lagging potentials. That is why, Coulomb's forces, with which the charges, which are at rest in the accelerated laboratory, interact, do not lie on the same straight line. But in the first place, this conclusion is not confirmed with experimentals, and in the second place, as we have seen, Coulomb's interaction of the charges, moving with uniform acceleration and calculated within framework of classical electrodynamics leads to the violation of laws of conservation of energy and impulse.

So the fact, that there is not any radiation of the charge, is not a reason to suppose, that the coordinate system, which is connected with the charge, is inertial. That fact only indicates, that all electromagnetic processes in this system will proceed in the same way as in the system, which is connected with the Earth.

But if we are not able to realize the inertial coordinate system, then all the systems become equal. In this case the acceleration loses its absolute sense and becomes a quantity, which is as relative as the velocity. So as to determine the intensity of the radiation, the quantity of the acceleration in expression (2) has to be chosen so as to take additional terms into account.

It is remarkable, that the concept of the equality of all coordinate systems has been completely realised in general theory of relativity. And what is more, if we managed to realise the inertial coordinate system, the general theory of relativity would turn out groundless.

5 The electrodynamics and the Principle of equivalence

We will try to remove the contradictions, which have been noticed before. First we will consider a charge moving freely in a gravitational field. So as the principle of equivalence will be true, this charge must not radiate. This conclusion seems to be rather logical. Indeed, it is supposed in classical physics, that there is a straight line, transpiercing through all the Universe and a charge, which is moving along it with constant velocity and does not radiate. Any deflection of such movement must be accompanied by the radiation. In classical physics only a ray of light must be a straight line. But light declines in a gravitational field. So one can get a straight line only when there is no any gravitational fields, i. e. in a limited spheres of space.

In the general theory of relativity a concept of geodesic line is introduced. It is a generalized notion of a straight line. It is a trajectory of the motion of a body, on which any forces do not act, except the gravitational ones. Such movement of a body in the general theory of relativity is inertial. So we can expect, that the free movement of a charge in the gravitational field will not be accompanied by radiation. That means, that in the coordinate system, freely moving in a gravitational field, the equations of Maxwell will have the same form, as in the inertial system.

Let us consider the charge q, which is at rest in the inertial coordinate system with a constant gravitational field. Evidently, vector $\vec{E}$ of such charge in any coordinate system will lie on a straight line, connected the charge and the point of observation. Indeed, let a charge e lie at a distance of charge q. The force, acting on the charge e from the side of the charge q, will be on a straight line, connected these charges. It is quite evidently, that the observer can move in any way, i. e. he can be in any coordinate system, but the direction of the force, acting on the charge q, will not change. Therefore field $\vec{E}$ of the charge q has to be described with expression

$$\vec{E} = \frac{q\,\vec{R}}{R^3} \cdot \frac{1 - \frac{v^2}{c^2}}{(1 - \frac{v^2}{c^2}\sin^2\theta)^{3/2}}, \tag{3}$$

which is to be true for this charge in any coordinate system. Here R is a distance from charge q to the point of the observation, θ is an angle between the direction of the motion of the charge and radius-vector $\vec{R}$, v is momentary velocity of the charge relative to the observer. Expression (3) describes the field of a uniformly moving charge. Its applicability in this case evident. Indeed, let there be an accelerated system and an inertial system, accompanying it. It is clear, that field $\vec{E}$ in the both systems will be identical at any moment for the observers.

For the charge q, which is at rest in a constant gravitational field, does not radiate then so as to fulfil the principle of equivalence, the charges, moving with uniform acceleration or uniformly moving round a circle, must not radiate either. Field $\vec{E}$ of such charges must also be described with expression (3), which is true for these charges in any coordinate system. (We consider only those charges, the quantity of the acceleration of which does not change during the period of observation, i. e. we except transitional processes). That means, that in the coordinate systems, which moves with an uniform acceleration or uniformly moves round a circle, Maxwell's equations has the same form as in the inertial systems. Under this assertion we mean the following: in the systems, moving with an uniform acceleration or uniformly moving round a circle, all electromagnetic processes will proceed in the same way as in the inertial systems with constant gravitational field.

But if the field $\vec{E}$ of the charges, moving with uniform acceleration or uniformly moving round a circle, is described with expression (3), there is no any violation of the laws of conservation by calculation of its interaction. Indeed, in this case the forces, which the charges, which were considered before, interact with, will be equal by the quantity and will be contrary directed.

The lack of the force of radiation friction during the motion with uniform acceleration becomes clear. For such charge does not radiate, so there will not be any violation of the balanse of energy.

How do the changes, which have been brought into electrodynamics, accord with the experimentals? It is the first question. And the second one — if the charge radiates, what value of the acceleration should we put down into the expression(2) to define intensity of the radiation.

Let us answer the second question first. We will consider a charge, oscillating in accordance with harmonic law relatively to inertial system K. Under inertial system K we mean here coordinate system, connected with the Earth. By such movement of the charge contradictions do not appear. So the charges, the motion of which can be brought to harmonic oscillations in system K, we will describe within the framework of classical electrodynamics. Now let the charge move straightforward with acceleration $w(t)$ in system K. If function $w(t)$ is periodic, we can expand it in trigonometric row

$$w(t) = w_0 + \sum_{n=1}^{\infty}(b_n \cos \omega nt + c_n \sin \omega nt), \tag{4}$$

where w_0 — constant of function $w(t)$, which is accorded with the movement of the charge with uniform acceleration and b_n and c_n — amplitudes of harmonic n. I. e. the function $w(t)$ can be expanded in a form of a sum of two items $w(t) = w_0 + w_1(t)$. Through $w_1(t)$ the sum of harmonics is designated in (4). For we think, that the uniformly accelerated charge does not radiate, then in the expression (2) for intensity of the radiation only value $w_1(t)$ has to be put in as an acceleration. If function $w(t)$ is not periodic, it has to be expanded into Fourier's integral. In this case the steady component w_0 is equal to zero. So we will put the complete value of the acceleration $w(t)$ into expression (2), i. e. the radiation of the charge will be defined in the same way as in classical dynamics. For in practice a charge can move straightforward only during a short period of time, its acceleration cannot be a periodic function. The radiation of such a charge therefore will completely correspond to parameters, which have been received within the framework of electrodynamics.

During the movement of a charge in a closed trajectory with a constant period of revolution, the value of its acceleration $w(t)$ can be also expanded in a trigonometric row (4). The steady component w_0 corresponds to the acceleration of the uniform moving round a circle here. So when we calculate the intensity of the radiation in expression (2), as an acceleration we will put in only the value $w_1(t)$. If function $w(t)$ is not periodic, we will put in the complete value of the acceleration.

The fact, that the charge, uniformly moving round a circle, does not have any radiation, does not contradict the existence of the synchrotron radiation. Indeed, the quantity of the acceleration of electrons in cyclical accelerators is not a periodic function. If we consider the movement of electron in the accelerator as periodic, we will receive the following resultat. The intensity of radiation I_1 on frequency of circulation w_1 (the first harmonic), calculated within the framework of classical electrodynamics, is proportionate to

$$w_0^2 + \frac{1}{2}(b_1^2 + c_1^2).$$

The intensity of the nth harmonic is proportionate to $\frac{1}{2}(b_n^2 + c_n^2)$. If we suppose, that the charge, uniformly moving round a circle, does not radiate, it will only bring us to the fact, that only the intensity of the first harmonic will decrease. In this case it must by proportionate to $\frac{1}{2}(b_1^2+c_1^2)$. The intensity of the rest harmonics will not change. As it is known from the electrodynamics, the great part of the radiation is concentrated in the range of frequencies $w \sim w_1 \left(\frac{\varepsilon}{mc^2}\right)^3$, where ε — is an energy of electron. On frequency w_1 a small part of energy is radiated. So when $\varepsilon = 50$ Mev $I_1/I \sim 10^{-8}$, where I is a complete intensity. It is necessary to take notice of the fact, that because of the influence of the conductive surfaces (the sides of the vacuum chamber of an accelerator) the intensity of the low frequency part of the radiation will decrease approximately $\left(\frac{r}{d}\right)^2$ times, where r is a radius of the orbit, and d is a distance from the electron beam to the conductive surface.

So it is clear, that it is practically impossible to notice such deflection in synchrotron radiation. We need a special experiment, which could help us to test the intensity of the radiation on frequency of revolution. When the charge uniformly moves round a circle, there must not be any radiation at all. In such way electron in atom can move in an stationary orbit. And as it is known, such electron does not radiate. This fact confirms the results we have received. On the other hand, we have received the explanations of the stability of Rutherford's atom, i. e. why electron does not radiate in a circular orbit.

6 Supplement

As it is known, the equations of physics, which are written in general covariance form, automatically satisfy the principle of equivalence. Therefore the general covariance expression for the intensity is not to violate the principle of equivalence. Let us receive such an expression. The charge is at rest in the inertial coordinate system. Its energy, which was radiated during period dt, is equal to

$$d\varepsilon = \frac{2e^2w^2}{3c^2}dt. \tag{5}$$

Complete radiated impulse in this coordinate system is equal to zero

$$d\vec{P} = 0. \tag{6}$$

In order to pass to an arbitary inertial coordinate system we will write down expressions (5) and (6) in four-dimensional form

$$dP^i = -\frac{2e^2}{3c}\frac{du^k}{ds}\cdot\frac{du_k}{ds}u^i ds. \tag{7}$$

So as to pass to general covariance expression for dP^i in (7) we will substitute usual differentiation for covariance. Then we will receive

$$dP^i = -\frac{2e^2}{3c}\left(\frac{du^k}{ds} + \Gamma^k_{l\,m}u^l u^m\right)\left(\frac{du_k}{ds} - \Gamma^i_{kl}u_i u^m\right)u^i ds. \tag{8}$$

Here $\Gamma^k_{l\,m}$ are symbols of Christoffel, which are

$$\Gamma^k_{i\,l} = \frac{1}{2}g^{k\,n}\left(\frac{\partial g_{n\,l}}{\partial x^m} + \frac{\partial g_{n\,m}}{\partial x^l} - \frac{\partial g_{l\,m}}{\partial x^n}\right), \tag{9}$$

where $g^{l\,m}$ is a fundamental tensor. Indexes take values 0, 1, 2, 3. Expression (8) is true in any coordinate system. For a body, moving freely in the gravitational field, it is

$$\frac{du^k}{ds} + \Gamma^k_{l\,m}u^l u^m = 0.$$

Therefore, the charge, moving in such way, does not radiate. We have received this conclusion before, when we were according electrodynamics with the principle of equivalence.

Now we will consider the charge, which is motionless in inertial coordinate system K with constant gravitational field. The acceleration of such charge in system K is zero, and the components of the 4-velocity are

$$u_0 = 1,\, u_1 = u_2 = u_3 = 0.$$

So it follows from (8), that the intensity of radiation of the charge in this case is

$$\frac{d\varepsilon}{dt} = \frac{2e^2c}{3}\Gamma^i_{0\,0}\Gamma^0_{i\,0}.$$

For a weak gravitational field components of tensor $g_{e\,m}$ are

$$g_{1\,1} \approx g_{2\,2} \approx g_{3\,3} \approx -1,\, g_{0\,0} = 1 + \frac{2\varphi}{c^2},\, g_{i\,k} \approx 0\,(i \neq k),$$

where φ is a gravitational potential. Taking the statical nature of the gravitational field into consideration, i. e. $\dfrac{\partial g_{k\,l}}{\partial x^0} = 0$, and that the components of tensor g^{ik} in this case have approximately the same values, as the components of tensor $g_{i\,k}$, we will receive, when α=1, 2, 3.

$$\Gamma^{\alpha}_{00} = \Gamma^{0}_{\alpha 0} = \frac{1}{2}\frac{\partial g_{00}}{\partial x^{\alpha}}.$$

Therefore, the intensity of radiation is equal to

$$\frac{d\varepsilon}{dt} = \frac{2e^2}{3c^3} \cdot \frac{\partial \varphi}{\partial x^{\alpha}} \cdot \frac{\partial \varphi}{\partial x_{\alpha}}. \tag{9}$$

For $\dot{\vec{v}} = -\nabla\varphi$, and acceleration $\dot{\vec{v}}$ in this case is the accelaration due to gravity g, then (9) will have the following form

$$\frac{d\varepsilon}{dt} = \frac{2e^2}{3c^3} g^2.$$

Thus, the charge, which is immovable on the Earth, must radiate. The intensity of its radiation is identical to the intensity of the charge, moving with acceleration g in empty space at a great distance away from any gravitating bodies. So the principle of equivalence is formally fulfilled. It is naturally for covariance expression (8), though it is clear, the charge, which is at rest in a gravitational field, does not charge.